Singlet and Triplet Superfluid Competition in a Mixture of Two-Component Fermi and One-Component Dipolar Bose gases

Ben Kain and Hong Y. Ling
Department of Physics and Astronomy, Rowan University, Glassboro, New Jersey 08028

We consider a mixture of two-component Fermi and (one-component) dipolar Bose gases in which both dipolar interaction and s-wave scattering between fermions of opposite spins are tunable. We show that in the long wavelength limit, the anisotropy in the Fermi-Fermi interaction induced by phonons of the dipolar condensate can strongly enhance the scattering in the triplet channel. We investigate in detail the conditions for achieving optimal critical temperature at which the triplet superfluid begins to compete with the singlet superfluid.

arXiv:1104.4759v2 [cond-mat.quant-gas] 27 Apr 2011

The ability to easily mix cold atoms of different species to form new quantum systems brings another exciting dimension to the study of ultracold atomic gases. A single-component Fermi gas only supports Cooper pairing with odd parities, such as p-wave pairing, which are typically strongly suppressed in accordance with Wigner's threshold law [1]. Mixing bosons induces an attractive interaction between fermions [2], which, as Efremov and Viverit [3] pointed out, raises the prospect of achieving p-wave superfluidity in a Fermi-Bose (FB) mixture. Recently, Dutta and Lewenstein [4] generalized the idea to a (2D) mixture involving dipolar bosons with the goal of realizing a superfluid with $p_x + ip_y$ symmetry whose excitations are non-Abelian anyons that are the building blocks for topological quantum computation [5], and Nishida [6] sought the same goal by mixing fermion gases of different species in different dimensions.

In this Letter, we consider a (3D) homogeneous mixture (with an effective volume V) between a *two-component* Fermi gas and a *dipolar* Bose gas, made up of two equally populated (balanced) hyperfine spin states ($|\uparrow\rangle$ and $|\downarrow\rangle$) of a non-dipolar fermionic atom of mass m_F, and a ground state of a bosonic molecule (or atom) of mass m_B with an induced dipole aligned along the external electric field direction z. The two pseudo spins provide fermions with the opportunity to pair not only via triplet (with odd parities) but also via singlet (with even parities) channels of interaction. This opens up the possibility of using this two-component FB model to emulate and explore pairing physics analogous to that in superfluid ^{3}He [7], which is known to be greatly enriched by the existence of an internal spin degree of freedom.

In the context of ultracold atomic physics, there has been a recent upsurge of activity in pursuing similar goals but with (3D) two-component dipolar Fermi gases [8, 9], motivated largely by recent rapid experimental advancement in achieving ultracold dipolar gases both in ^{40}K -^{87}Rb polar molecules [10], and in Cr [11] and spin-1 Rb atoms [12]. Such studies [8, 9] represent a generalization of earlier work [13, 14] aimed at exploiting a $d_{r^2-3z^2}$ -type of anisotropy in dipole-dipole interactions for enhancing triplet pairing in single-component dipolar Fermi systems.

The induced Fermi-Fermi interaction mediated by a dipolar condensate is also anisotropic in nature and thus opens up a new avenue for studying superfluids with unusual pairings. The progress in this area has so far been limited, to the best of our knowledge, to a single-component model in a 2D geometric setting [4]. In contrast, the present work expands such studies to a 3D two-component mixture, where both the dipolar interaction between bosons and the s-wave scattering between fermions of opposite spins are independently tunable, and seeks to use it as a model to explore the physics that are currently being hotly pursued in two-component dipolar Fermi gas systems [9]. In this Letter, we study in detail the anisotropic nature of the 3D induced interaction, and particularly how one should prepare a two-component FB mixture in order to maximize the opportunity this induced interaction affords for raising critical temperatures at which phases of different parities begin to compete.

To begin with, we model our system with a (grand canonical) Hamiltonian $\hat{H} = \hat{H}_B + \hat{H}_{BF} + \hat{H}_F$:

$$\hat{H}_B = \sum_{\mathbf{k}} \left(\xi_{\mathbf{k},B} \equiv \epsilon_{\mathbf{k},B} - \mu_B\right) \hat{b}^\dagger_{\mathbf{k}} \hat{b}_{\mathbf{k}} + (2V)^{-1} \times \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \left[U_{BB} + U_{DD}(\mathbf{q})\right] \hat{b}^\dagger_{\mathbf{k}+\mathbf{q}} \hat{b}^\dagger_{\mathbf{k}'-\mathbf{q}} \hat{b}_{\mathbf{k}'} \hat{b}_{\mathbf{k}}, \quad (1)$$

$$\hat{H}_{BF} = U_{BF} (2V)^{-1} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\sigma} \hat{a}^\dagger_{\sigma,\mathbf{k}} \hat{a}_{\sigma,\mathbf{k}+\mathbf{q}} \hat{b}^\dagger_{\mathbf{k}'} \hat{b}_{\mathbf{k}'-\mathbf{q}}, \quad (2)$$

$$\hat{H}_F = \sum_{\mathbf{k},\sigma} \left(\xi_{\mathbf{k},F} \equiv \epsilon_{\mathbf{k},F} - \mu_F\right) \hat{a}^\dagger_{\mathbf{k},\sigma} \hat{a}_{\mathbf{k},\sigma} + (2V)^{-1} \times \sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\sigma,\sigma'} U_{\sigma\sigma'}(\mathbf{q})\, \hat{a}^\dagger_{\mathbf{k}+\mathbf{q},\sigma} \hat{a}^\dagger_{\mathbf{k}'-\mathbf{q},\sigma'} \hat{a}_{\mathbf{k}',\sigma'} \hat{a}_{\mathbf{k},\sigma}, \quad (3)$$

where $\hat{b}_{\mathbf{k}}$ ($\hat{a}_{\sigma,\mathbf{k}}$) is the field operator for annihilating a boson (a fermion of spin σ) of kinetic energy $\epsilon_{\mathbf{k},B} = \hbar^2 k^2/2m_B$ $\left(\epsilon_{\mathbf{k},F} = \hbar^2 k^2/2m_F\right)$ and chemical potential μ_B (μ_F). The low temperature physics of the mixture under consideration arises from the interplay between short- and long-range two-body interactions. The former is dominated by s-wave scattering characterized with strengths: $U_{BB} = 4\pi\hbar^2 a_{BB}/m_B$, $U_{FF} = 4\pi\hbar^2 a_{FF}/m_F$, $U_{BF} = 4\pi\hbar^2 a_{BF}/m_{BF}[\equiv 2m_B m_F/(m_B + m_F)]$, and $U_{\sigma\sigma'}(\mathbf{q}) = U_{FF}\delta_{\sigma',-\sigma}$, where a_{BB}, a_{BF} and a_{FF} are

the related scattering lengths, and the Kronecker-δ function in $U_{\sigma\sigma'}(\mathbf{q})$ limits the fermionic s-wave interactions to fermions of opposite spins. The latter is the dipole-dipole interaction (restricted to bosons) given by $U_{DD}(\mathbf{q}) = 8\pi d^2 P_2(\cos\theta_{\mathbf{q}})/3$ in momentum space, with d^2 the dipolar interaction strength, $P_2(x) = (3x^2-1)/2$ the second-order Legendre polynomial, and $\theta_{\mathbf{q}}$ ($\phi_{\mathbf{q}}$) the polar (azimuthal) angle of vector $\mathbf{q}$.

In the low temperature regime considered in the present Letter, bosons are virtually all condensed to the zero-momentum mode and a straightforward application of the Bogoliubov approximation, in which $\hat{b}_{\mathbf{k}=0}$ is treated as a c-number $b_{\mathbf{k}=0}$, yields a well-known picture of the bosonic system described by Eq. (1): it consists of a collection of phonon modes with the Bogoliubov dispersion relation $E_{\mathbf{k}} = v_B \hbar k\sqrt{1+(\xi_B k)^2 + 2\varepsilon_{dd}P_2(\cos\theta_{\mathbf{k}})}$ [15] and a homogeneous dipolar condensate with density $n_B = |b_{\mathbf{k}=0}|^2$ ($\mu_B = n_B U_{BB}$), which is stable against collapse provided $\varepsilon_{dd} < 1$, above which phonons with $k \to 0$ acquire imaginary frequencies. Here, $\varepsilon_{dd} = 4\pi d^2/(3U_{BB})$ [16] measures the strength of the dipolar interaction relative to the s-wave interaction, $v_B = \sqrt{n_B U_{BB}/m_B}$ is the phonon speed, and $\xi_B = \hbar/\sqrt{4 m_B n_B U_{BB}}$ is the healing length. Integrating away the phonon degrees of freedom [2] leads to an effective Fermi system described by the same Hamiltonian as Eq. (3), except that $U_{\sigma\sigma'}(\mathbf{k}) = U_{FF}\delta_{\sigma',-\sigma} + U_{ind}(\mathbf{k})$, where

$$U_{ind}(\mathbf{k}) = -\frac{U_{BF}^2/U_{BB}}{1+(\xi_B k)^2 + 2\varepsilon_{dd}P_2(\cos\theta_{\mathbf{k}})} \tag{4}$$

is the phonon-induced Fermi-Fermi interaction in the static limit [2, 3]. As can be seen, the induced interaction depends on the dipole orientation differently than the direct dipole-dipole interaction and therefore provides an alternative model for the exploration of spin singlet and triplet paring.

Typical BCS mean-field theory proceeds with the introduction of the matrix representation for the BCS order parameter in the uncoupled spin space: $\Delta_{\sigma'\sigma}(\mathbf{k}) = \sum_{\mathbf{k}'} U_{\sigma\sigma'}(\mathbf{k}-\mathbf{k}')\langle \hat{a}_{-\mathbf{k}',\sigma'}\hat{a}_{\mathbf{k}',\sigma}\rangle$. The notion of spin singlet and triplet pairing emerges when one moves from uncoupled to coupled spin space spanned by a spin singlet $|S=0, M=0\rangle$ and triplet $|S=1, M=-1,0,+1\rangle$, which are antisymmetric and symmetric with respect to the spin exchange, respectively, where M is the z projection of the total spin S. As one may easily verify, the gap parameter, $\Delta^s(\mathbf{k}) = (\Delta_{\uparrow\downarrow}(\mathbf{k}) - \Delta_{\downarrow\uparrow}(\mathbf{k}))/2$ associated with the singlet ($S=0$) state is an even function of $\mathbf{k}$, while the three gap parameters, $\Delta^{t,x}(\mathbf{k}) = (\Delta_{\downarrow\downarrow}(\mathbf{k}) - \Delta_{\uparrow\uparrow}(\mathbf{k}))/2$, $\Delta^{t,y}(\mathbf{k}) = (\Delta_{\downarrow\downarrow}(\mathbf{k}) + \Delta_{\uparrow\uparrow}(\mathbf{k}))/2i$, and $\Delta^{t,z}(\mathbf{k}) = (\Delta_{\uparrow\downarrow}(\mathbf{k}) + \Delta_{\downarrow\uparrow}(\mathbf{k}))/2$, associated with the triplet ($S=1$) states are odd functions of $\mathbf{k}$, in accordance with Fermi statistics, where use of $\Delta_{\alpha\beta}(\mathbf{k}) = \Delta^s(\mathbf{k})\, i(\sigma_y)_{\alpha\beta} + \sum_{u=x,y,z}\Delta^{t,u}(\mathbf{k})\, i(\sigma_u\sigma_y)_{\alpha\beta}$, a convention in the study of superfluid ^{3}He [7], has been made, with σ_u being the usual Pauli matrices.

To highlight the dominant physics, we ignore the Fermi surface deformation due to the anisotropy of the Fermi-Fermi interaction [17] and consider two-body scattering up to the level of the Born approximation [13, 14], both of which hold in the weakly interacting regime where $n_F U_{BF}^2/U_{BB} \ll \epsilon_F =$ Fermi energy. At critical temperatures where the gaps are small, one can ignore the nonlinear coupling between parings of different parities so that the critical temperatures can be estimated by a set of linearly coupled gap equations [9, 14]:

$$\Delta(\mathbf{k}) = -V^{-1}\sum_{\mathbf{k}'} U(\mathbf{k},\mathbf{k}')\, K(k')\, \Delta(\mathbf{k}), \tag{5}$$

with the understanding that $\Delta(\mathbf{k}) = \Delta^s(\mathbf{k})$ and $U(\mathbf{k},\mathbf{k}') = U^s(\mathbf{k},\mathbf{k}')$ for singlet pairing and $\Delta(\mathbf{k}) = \Delta^{t,u}(\mathbf{k})$ and $U(\mathbf{k},\mathbf{k}') = U^t(\mathbf{k},\mathbf{k}')$ for triplet pairing, where $U^s(\mathbf{k},\mathbf{k}') = U_{FF} + [U_{ind}(\mathbf{k}-\mathbf{k}') + U_{ind}(\mathbf{k}+\mathbf{k}')]/2$ and $U^t(\mathbf{k},\mathbf{k}') = [U_{ind}(\mathbf{k}-\mathbf{k}') - U_{ind}(\mathbf{k}+\mathbf{k}')]/2$ are the singlet and triplet potentials that are even and odd functions of both $\mathbf{k}$ and $\mathbf{k}'$, respectively, and $K(k) = \tanh(\beta\xi_k/2)/(2\xi_k) - 1/(2\epsilon_k)$. In arriving at Eq. (5), we have applied the standard procedure to renormalize the contact interaction [2] and a similar procedure (but expressed in terms of vertex functions [14]) to renormalize the dipolar interaction.

Making decompositions: $\Delta(\mathbf{k}) = \sum_l \Delta_l(k)\, Y_l^0(\hat{\mathbf{k}})$ and $U(\mathbf{k},\mathbf{k}') = 4\pi\sum_{l,l',m} U_{lm,l'm}(k,k')\, Y_{lm}^*(\hat{\mathbf{k}}) Y_{l'm}(\hat{\mathbf{k}}')$, in which the azimuthal symmetry of the interaction (4) has been explicitly incorporated, we cast Eq.(5) into the form containing only the radial coordinate:

$$\Delta_l(k) = -\sum_{l'}\int \frac{k'^2}{2\pi^2} K(k')\, U_{l0,l'0}(k,k')\, \Delta_{l'}(k')\, dk', \tag{6}$$

where $Y_{lm}(\hat{\mathbf{k}})$ are spherical harmonic functions. In the low temperature limit $k_B T/\epsilon_F \ll 1$, $k^2 K(k)$ is small virtually everywhere except around the Fermi momentum k_F [$= (3\pi^2 n_F)^{1/3}$] where it is sharply peaked compared to other momentum distributions, and the critical temperature can be estimated, to a good approximation, by the equation for the gap parameter, $\Delta_l \equiv \Delta_l(k_F)$, at the Fermi surface

$$\Delta_l = N(\epsilon_F)\ln\frac{\pi k_B T}{8\epsilon_F e^{\gamma-2}}\sum_{l'} U_{l,l'}\Delta_{l'} \tag{7}$$

where $\gamma = 0.577$ is Euler's constant, and $U_{l,l'} \equiv U_{l0,l'0}(k_F,k_F)$ is an element of the interaction matrix U: $U_{l,l'}^s = U_{FF}\delta_{l,0}\delta_{l',0} + 0.5[1+(-1)^{l'}]U_{l,l'}^{ind}$ and $U_{l,l'}^t = 0.5[1-(-1)^{l'}]U_{l,l'}^{ind}$, with

$$U_{l,l'}^{ind} = 2\pi\int\int\left[\int U_{ind}(k_F\hat{\mathbf{k}} - k_F\hat{\mathbf{k}}')\, d(\phi_{\mathbf{k}} - \phi_{\mathbf{k}'})\right]\times Y_{l0}(\cos\theta_{\mathbf{k}})\, Y_{l'0}(\cos\theta_{\mathbf{k}'})\, d(\cos\theta_{\mathbf{k}})\, d(\cos\theta_{\mathbf{k}'}). \tag{8}$$

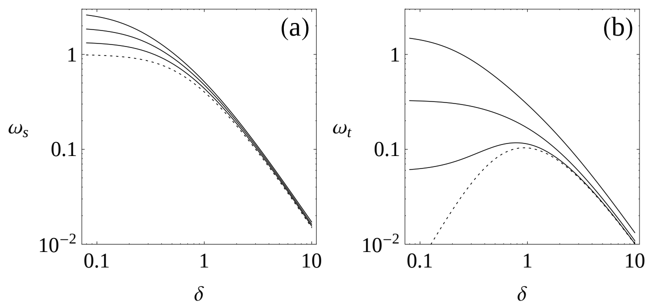

FIG. 1. The scaled eigenvalue ω_s for the singlet state (a) and ω_t for the triplet state (b) as functions of δ for different ε_{dd}. In both figures, the dotted curves are for the nonpolar case ($\varepsilon_{dd} = 0$) and the solid curves from bottom to top are for (a) $\varepsilon_{dd} = 0.6$, 0.8 and 0.9, and (b) ε_{dd} =0.1, 0.5 and 0.9.

In contrast to the dipole-dipole interaction, which couples Δ_l only to $\Delta_{l\pm2}$, the dipole induced interaction in Eq. (4) can, in principle, couple Δ_l to any $\Delta_{(l+2n)\geq0}$ with n being an integer according to Eq. (8). As expected, the singlet and triplet pairings are decoupled, containing all the even and odd partial wave components, respectively.

To solve Eq. (7), we move into a primed space in which the interaction matrix $U' = MUM^\dagger$ is diagonalized via a unitary transformation M so that $U'_{n,n'} = -\omega_n\left(U_{BF}^2/U_{BB}\right)\delta_{n,n'}$,where ω_n is the eigenvalue scaled to $-U_{BF}^2/U_{BB}$. In this primed space, Eqs. (7) are decoupled, leading, immediately, to the critical temperature

$$T_n = \frac{8\epsilon_F e^{\gamma-2}}{\pi k_B}\exp\left[-\frac{2}{\omega_n(\delta)\lambda}\right], \tag{9}$$

for the nth channel, in which the order parameter $\Delta(\mathbf{k}) \propto \sum_l M^*_{n,l}Y_{l0}(\hat{\mathbf{k}})$ becomes a superposition of different partial waves with different angular momenta, where

$$\delta = \xi_B k_F = k_F/\left(4\sqrt{\pi n_B a_{BB}}\right), \tag{10}$$

$$\lambda = 2N(\epsilon_F)\frac{U_{BF}^2}{U_{BB}} = \frac{4}{\pi}\frac{m_B m_F}{m_{BF}^2}\frac{a_{BF}^2}{a_{BB}}k_F. \tag{11}$$

As the temperature is lowered, the most favorable superfluid phases to be realized correspond to those channels with the strongest attractive interactions (the highest positive ω_n).

Figure 1 shows how the strongest (attractive) interactions in the singlet and triplet channels, ω_s and ω_t, change with δ for different ε_{dd} when $U_{FF} = 0$. The dotted curves, $\omega_s = (2\delta)^{-2}\ln[1+(2\delta)^2]$ and $\omega_t = 2(2\delta)^{-2}\left\{\ln[1+(2\delta)^2][(2\delta)^{-2}+2^{-1}]-1\right\}$, represent the corresponding interactions in mixtures with nonpolar molecules ($\varepsilon_{dd} = 0$) [3, 18], where, as δ reduces, the triplet interaction begins to decrease to zero after reaching its peak around $\delta \approx 1$ as opposed to the singlet interaction, which increases monotonically. The introduction of a dipolar condensate to a fermion gas adds to the denominator of Eq. (4) an anisotropic term $2\varepsilon_{dd}P_2(\cos\theta_{\mathbf{k}})$, which plays an increasingly important role compared to the isotropic contribution $1+\delta^2$ in the small δ region, around which significant changes are observed to take place in Fig. 1. The most striking development happens, however, in the triplet interaction [Fig. 1(b)] which not only asymptotes to a finite value in the limit $\delta \to 0$, in clear defiance of Wigner's threshold law, but also increases with ε_{dd} for a given δ far more dramatically than the singlet interaction [Fig. 1(a)]. This provides concrete evidence that the use of a dipolar BEC can indeed significantly enhance scattering in the triplet channel compared to the singlet channel, which has been the key motivation behind the present proposal for achieving the triplet superfluid.

To prepare a system with small δ, we must employ a Bose component with a relatively small (high) healing length (density). This often means that the system separates into a mixed phase with densities (n_{F1}, n_{B1}) and a pure Fermi phase with densities $(n_{F2}, n_{B2} = 0)$, the only phase separation scenario that involves a mixed phase [19, 20]. (A complete separation between fermions and bosons requires much higher densities than considered in the present work.) The mixed phase must share the same chemical and thermodynamical potentials with the pure phase. This consideration leads to

$$U_{BF}n_{B1} + An_{F1}^{2/3} = An_{F2}^{2/3},$$

$$-U_{BB}n_{B1}^2/2 - U_{BF}n_{B1}n_{F1} - 2An_{F1}^{5/3}/5 = -2An_{F2}^{5/3}/5,$$

from which one finds

$$n_{B1} = An_{F1}^{2/3}\left(y^2-1\right)/U_{BF}, \tag{12}$$

where $A = \left(3\pi^2\right)^{2/3}\hbar^2/(2m_F)$ and $y = (n_{F2}/n_{F1})^{1/3}$ is the solution to the cubic equation

$$-15(y+1)^2/\lambda + 8y^3 + 16y^2 + 24y + 12 = 0, \tag{13}$$

(see Ref. [19] for details). All previously derived formulas concerning the critical temperature are directly applicable to the mixed phase upon substitution of n_B with n_{B1} and n_F with n_{F1}.

The optimal triplet superfluid temperature T_t is always found to occur in the mixed state. An example in which $m_F = 6u$, $m_B = 127u$, and $a_{BB} = 250a_0$ (with u the atomic mass and a_0 the Bohr radius) is given in Fig. 2 (a), which illustrates how T_t and the required a_{BF} change with n_{F1} for different ε_{dd}. In arriving at Fig. 2 (a), we used Eq. (9) to construct, for a given set of n_{F1} and ε_{dd}, temperature T_t as a function of λ by solving for the required a_{BF}, n_{B1}, and δ simultaneously from Eqs. (10) - (13). The optimal T_t corresponds to the peak temperature at some $\lambda = \lambda_{\text{peak}}$. In Fig. 2 (a), we find that $\lambda_{\text{peak}} \approx 0.6$ and $\delta_{\text{peak}} \approx 0.25$ with small variation for

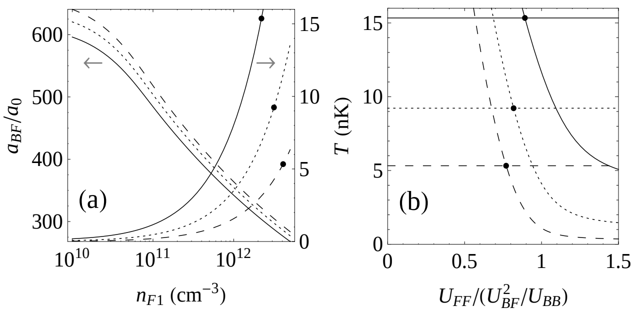

FIG. 2. (a) The optimal T_t and the required a_{BF} as functions of n_{F_1} for different ϵ_{dd} under conditions that $m_F = 6u$, $m_B = 127u$ and $a_{BB} = 250a_0$. (b) illustrates how T_s (curves) can be made degenerate with T_t (horizontal lines) by changing U_{FF} while fixing all the other parameters to those represented by the black dots in (a) where n_{B1} have reached 5×10^{14} cm^{-3}. In both figures, $\epsilon_{dd} = 0.9$ for solid curves, $\epsilon_{dd} = 0.85$ for dotted curves, and $\epsilon_{dd} = 0.8$ for dashed curves.

different values of n_{F1} and ε_{dd}. As can be seen from the solid curves ($\varepsilon_{dd} = 0.9$), a temperature about 15 nK can be achieved in a mixed phase (marked with a black dot) with densities ($n_{B1} = 5 \times 10^{14}$ cm^{-3}, $n_{F1} = 2.17 \times 10^{12}$ cm^{-3}) and $a_{BF} = 304a_0$.

As to the singlet superfluid temperature T_s, it depends on the contact interaction U_{FF} [or $(k_F a_{FF})^{-1}$], which in our model is made magnetically tunable via Feshbach resonance. Thus, in principle, the interaction in the singlet channel can be made degenerate to that in the triplet channel by tuning both the dipolar interaction (ε_{dd}) with an electric field and the s-wave scattering length (a_{FF}) with a magnetic field. This opens up the possibility of studying phase competition between singlet and triplet superfluids, a recurring theme in current studies concerning two-component dipolar Fermi gases. Figure 2 (b) illustrates how T_s (curves) can be made to cross T_t (horizontal lines) by changing U_{FF} for different ε_{dd}. In contrast to the pure two-component dipolar Fermi gas model, where, due to the average of the dipolar interaction over all the directions being zero, the singlet interaction is always less attractive than the triplet interaction in the absence of U_{FF}, and U_{FF} must be tuned to the negative side of the Feshbach resonance in order to make T_s comparable to T_t [9], the singlet interaction in our model is more attractive than the triplet interaction in the absence of U_{FF} [Fig. 1], and consequently only when U_{FF} is tuned to the positive side of the Feshbach resonance, can T_s be brought down to the level of T_t [Fig. 2 (b)]. An estimate based on $U_{FF}/\left(U_{BF}^2/U_{BB}\right) \approx 1.08$ at the crossing of the two solid lines ($\varepsilon_{dd} = 0.9$) in Fig. 2 (b) indicates that $T_s = T_t$ when $(k_F a_F)^{-1} = 2.03$.

In summary, we have investigated the optimal conditions for achieving the coexistence between singlet and triplet superfluids in a two-component FB mixture with a dipolar condensate. We have found that T_s can be made degenerate to T_t at a temperature 10^7 orders of magnitude higher than 10^{-6} nK (not shown), the optimal temperature achievable under a similar set of fixed parameters for a two-component FB mixture with nondipolar bosons. Just as mixing nonlinear waves has been an important means for creating coherent sources of laser light, mixing cold atoms is expected to play an increasingly more important role in creating new quantum gases (or liquids) in the coming years as the field of ultracold atomic physics continues to mature. The present study reinforces the notion that mixing fermions with dipolar bosons adds another exciting dimension in the pursuit of quantum systems with new and novel properties.

H. Y. L. acknowledges the support from the US National Science Foundation and the US Army Research Office.

[1] L. D. Landau and E. M. Lifshitz, Quantum Mechanics: Non-Relativitic Theory (Pergamon Press, Oxford, 1977, 3rd ed.)
[2] M. J. Bijlsma, B. A. Heringa, and H. T. C. Stoof, Phys. Rev. A **61**, 053601 (2000).
[3] D. V. Efremov and L. Viverit, Phys. Rev. B 65, 134519 (2002).
[4] O. Dutta and M. Lewenstein, Phys. Rev. A **81**, 063608 (2010).
[5] C. Nayak, et al., Rev. Mod. Phys. 80, 1083 (2008).
[6] Yusuke Nishida, Annals Phys. **324**, 897 (2009).
[7] D. Vollhardt and P. Wölfle, The superfluid phases of Helium 3, (Taylor and Francis, New York, 1990).
[8] K. V. Samokhin and M. S. Mar'enko, Phys. Rev. Lett. 197003 (2006);
[9] T. Shi, et al., Phys. Rev. A 82, 033623 (2010); C. Wu and J. E. Hirsch, Phys. Rev. B **81**, 020508 (R) (2010); R. Liao and J. Brand, Phys. Rev. A **82**, 063624 (2010).
[10] S. Ospelkaus, et al., Nature Phys. 4, 622 (2008); K. K. Ni, et al., Science **322**, 231 (2008). S. Ospelkaus, et al., Faraday Discuss. **142**, 351 (2009).
[11] J. Stuhler et al, Phys. Rev. Lett. **95**, 150406 (2006).
[12] M. Vengalattore, et al., Phys. Rev. Lett. **100**, 170403 (2008).
[13] M. Marinescu and L. You, Phys. Rev. Lett. **81**, 4596 (1998); L. You and M. Marinescu, Phys. Rev. A **60**, 2324 (1999); S. Yi and L. You, Phys. Rev. A 61, 041604 (2000).
[14] M. A. Baranov, et al., Phys. Rev. A **66**, 013606 (2002); M. A. Baranov, L. Dobrek, and M. Lewenstein, Phys. Rev. Lett. **92**, 250403 (2004);M. A. Baranov et al., Phys. Rev. A **77**, 061603 (2008).
[15] K. Góral, K. Rzążewski, and T. Pfau, Phys. Rev. A **61**, 051601 (2000); S. Giovanazzi, A. Görlitz, and T. Pfau, Phys. Rev. Lett. **89**, 130401 (2002).
[16] D. H. J. O'Dell, S. Giovanazzi, and C. Eberlein, Phys. Rev. Lett. 92, 250401 (2004); C. Eberlein, S. Giovanazzi, and D. H. J. O'Dell, Phys. Rev. A 71, 033618 (2005).
[17] T. Miyakawa, T. Sogo, and H. Pu, Phys. Rev. A 77,

061603(R) (2008); T. Sogo, et al., N. J. Phys. 11, 055017 (2009); B. M. Fregoso, et al., New J. Phys. 11, 103003 (2009).

[18] A. Bulgac, M. M Forbes, and A. Schwenk, Phys. Rev. Lett. **97**, 020402 (2006).

[19] L. Viverit,C. J. Pethick, and H. Smith, Phys. Rev. A **61**, 053605 (2000).

[20] Michael Fodor and Hong Y. Ling, Phys. Rev. A **82**, 043610 (2010).